# Investigation of Low-energy Antinucleon-nucleus Interactions for an Experiment to Search for Neutron-antineutron Oscillations with Ultracold Neutrons

IGORR22

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Hiroyuki Fujioka (Institute of Science Tokyo)

# Background: Baryon Asymmetry of the Universe

Image credit: James Webb Telescope

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- **Baryon asymmetry**: dominance of matter over antimatter in the universe
  - Cosmologically observed baryon-antibaryon asymmetry:

$$\eta = (N_B - N_{\bar{B}})/N_\gamma$$

Cosmological observation: $\eta_{\mathrm{obs}} \sim 10^{-10}$, theory: $\eta_{theory} \sim 10^{-20}$ → **discrepancy by$10^{10}$**

W. Buchmüller et al., Nucl. Phys. B 643, 367 (2002); L. Canetti et al., NJP **14**, 095012 (2012)

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- **Sakharov's conditions (1967):** requirements for a matter-dominant universe
  - Departure from thermal equilibrium in early universe
  - Violation of baryon number ($B = 1\ (p, n), B = -1\ (\bar{p}, \bar{n})$)
  - Violation of *C* (charge) and *CP* (charge-parity) symmetries

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*One of the biggest mysteries in today's particle physics*

- **Baryon number violation:** candidate processes
  - Proton decays ($p \to e^+\pi^0, p \to \bar{\nu}K$ , etc): |ΔB|=1, $\tau_p > 9 \times 10^{29}$ yr
  - Neutron-antineutron oscillations ($n \leftrightarrow \bar{n}$): |ΔB|=2, $\tau_{n\bar{n}} > 4.7 \times 10^8$ s

    Different sensitivities to Grand Unified Theories (GUTs)

    Opportunity to search for $n$-$\bar{n}$ oscillations at the New Research Reactor

Particle Data Group https://pdg.lbl.gov/ (2025)



Image credit: James Webb Telescope

# Outline

1. Context: Neutron-antineutron oscillation
   - Background
   - Previous experiment
   - Uniqueness of the new proposal and the keys
2. Antinucleon-nucleus interactions
   - X-ray spectroscopy of antiprotonic atoms
   - Low-energy antineutron scattering

# Previous work and future plans

- Current best limits:
  - For free neutrons: $\boldsymbol{\tau_{n\bar{n}}} > \mathbf{8.6} \times \mathbf{10^7}$s (CL 90%)
  - For bound neutrons ($^{16}$O): $\boldsymbol{\tau_{n\bar{n}}} > \mathbf{4.7} \times \mathbf{10^8}$s (CL 90%)

M. Baldo-Ceolin *et al.*, Z. Phys. C **63**, 409 (1994).
K. Abe *et al.* Phys. Rev. D **103**, 012008 (2021).

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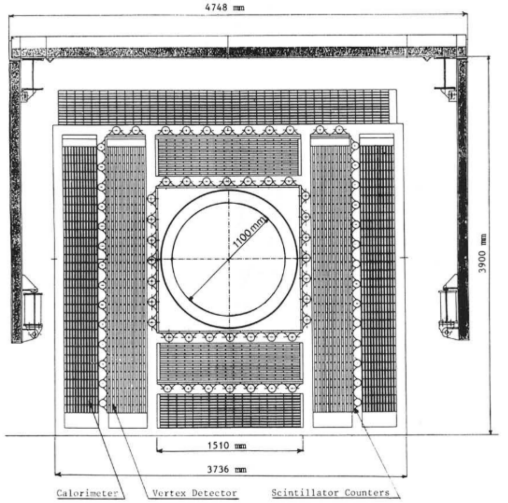
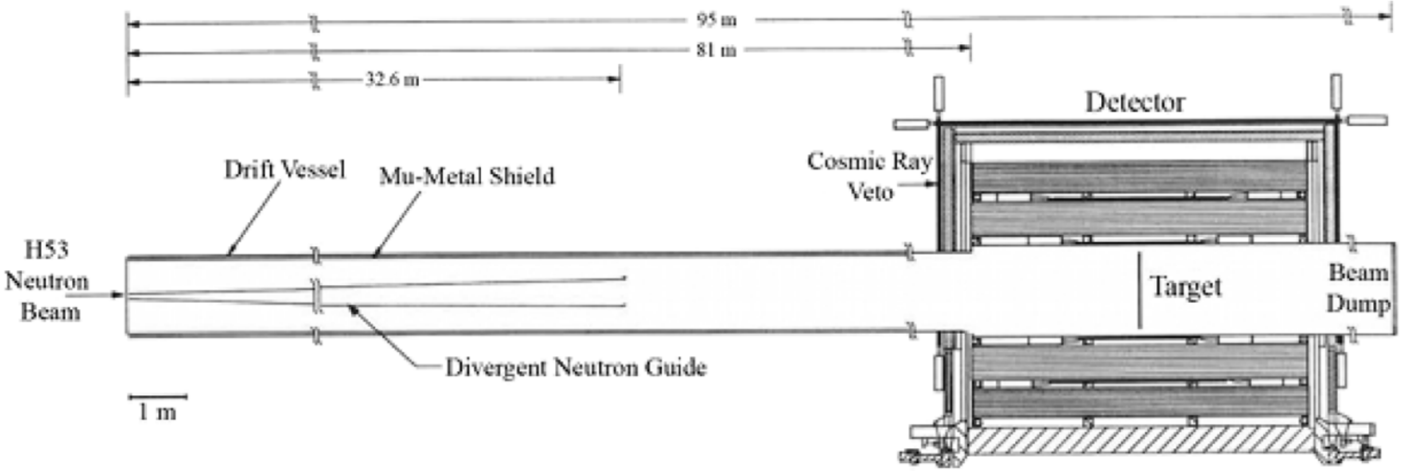
- Experiment at the Institut Laue-Langevin (HFR) (1994)
  - Searched for $\boldsymbol{\bar{n}}$ by annihilation to pions (130 um-thick 110 cm diameter graphite film as annihilation target, $\varepsilon_{det}$=0.52)
  - Cold neutron flux: ~2 meV, 1.25 x$10^{11}$ n/s
  - Flight path: 76.5 m, 0.1 s
  - Magnetic field: < 10 nT, pressure: < 1E-2 Pa
  - Accumulated run time: $Y_{\bar{n}}$ = 2.4 x$10^7$ s
    → $\boldsymbol{\tau_{n\bar{n}}} > \mathbf{8.6} \times \mathbf{10^7}$s (CL 90%)

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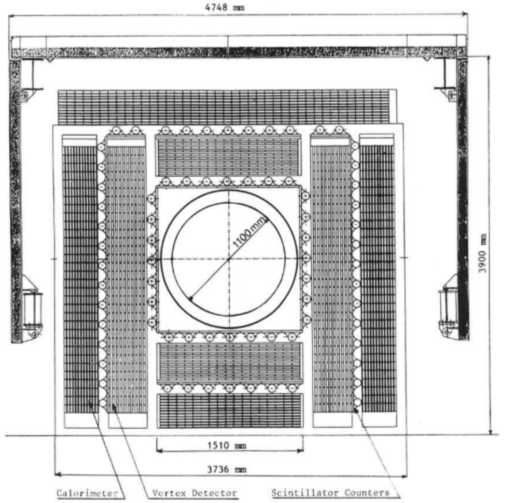
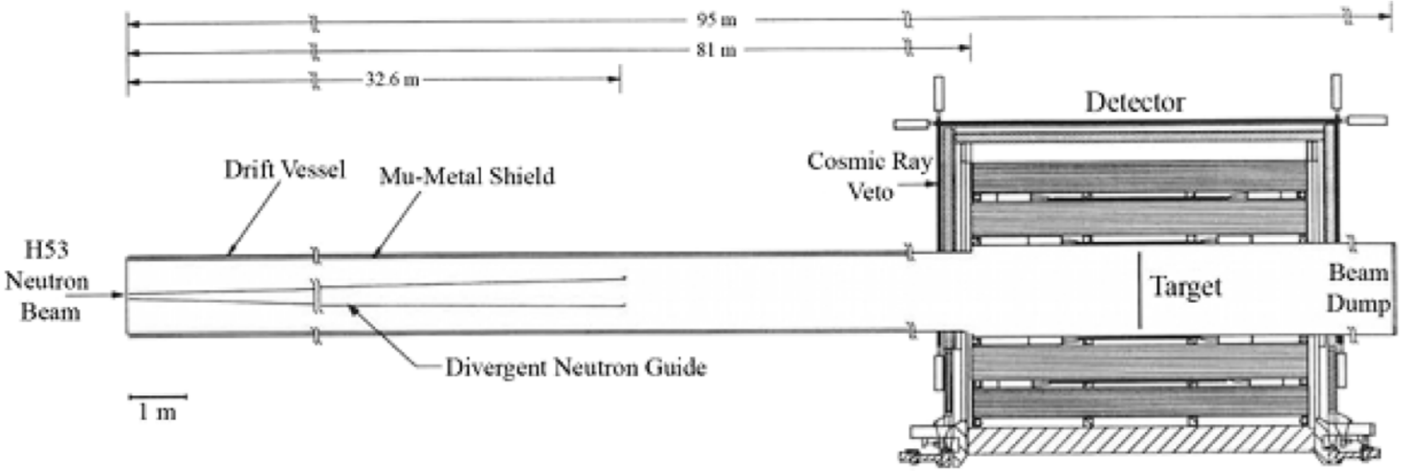
- **Future projects:**
  - European Spallation Source: free neutrons, x32 improved statics
  - New Research Reactor in Japan: with stored **ultracold neutrons (UCNs)**

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# Fermi potential and ultracold neutrons

- At low energy, neutron-nucleus scattering represented by the s-wave □ Fermi potential

**Fermi potential**

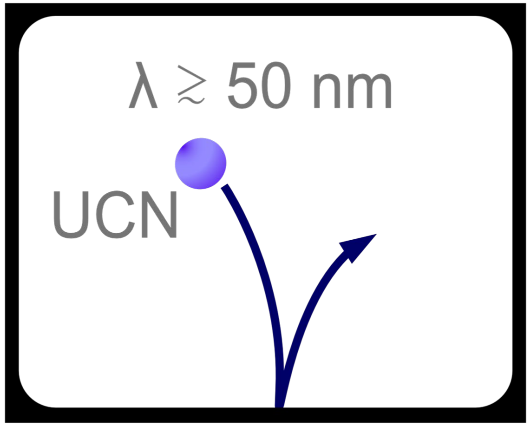
$$V_{\mathrm{F}} = \frac{2\pi\hbar^2 N\boxed{b}}{m_n} \left( = \frac{2\pi\hbar^2 N a}{\mu} \right)$$

- *N*: number density of nuclei
- <u>*b* (or *a*): scattering length</u>
- $m_n$: neutron mass (μ: reduced mass)
- For alloy/compound (e.g. NiP):
  $b$ □ $(b_1 n_1 + b_2 n_2)/(n_1 + n_2)$

| Material | s-wave scattering length *b* (fm) | Fermi potential $V_F$ (neV) |
|---|---|---|
| Be | 7.778−0.002 *i* | 249.01−0.068 *i* |
| C | 6.648−0.0009732 *i* | 195.3−0.029 *i* |
| P | 5.130−0.0478 *i* | 47.53−0.443 *i* |
| Ni | 10.30−1.248 *i* | 243.5−29.52 *i* |
| NiP | — | 213.2−0.0229 *i* |

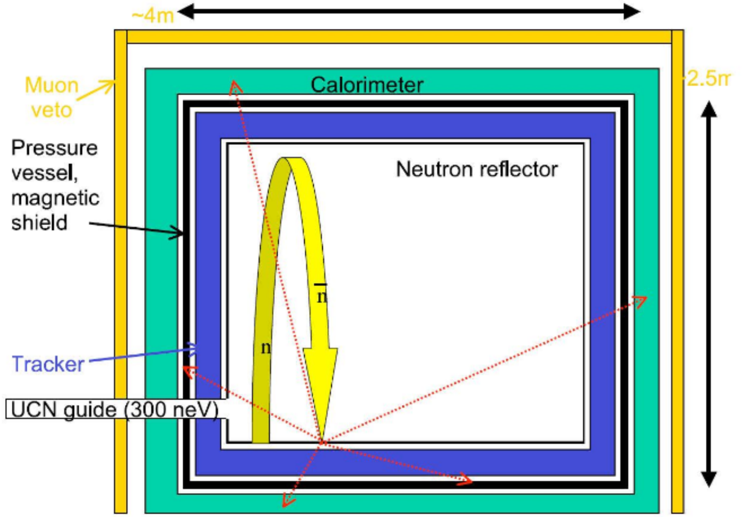
Measured by neutron interferometry with ~0.1% precision

- ***<u>Ultracold neutrons (UCNs):</u>*** neutrons with energies ≲ 300 neV
  □ Can be stored for time **~100 s** in a material vessel

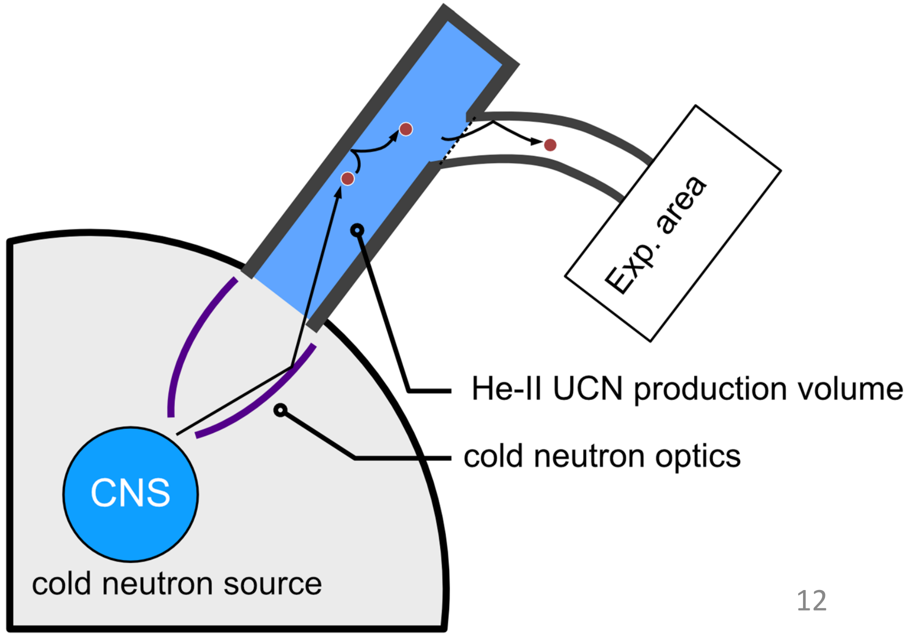


# Proposed Experiment at the New Research Reactor

- Search for an $n$-$\bar{n}$ oscillation with stored UCNs
  - Significant gain in observation time: **0.1 s → ~100 s**
  - Intense UCN source required but feasible: ~**$10^7$-$10^8$ UCN/s**
    - Technologies developed at TRIUMF (→ Next talk)
    - Opportunity to design the UCN source from early stage of cold neutron source design
  - High up-time of a reactor → allows a few years of data collection

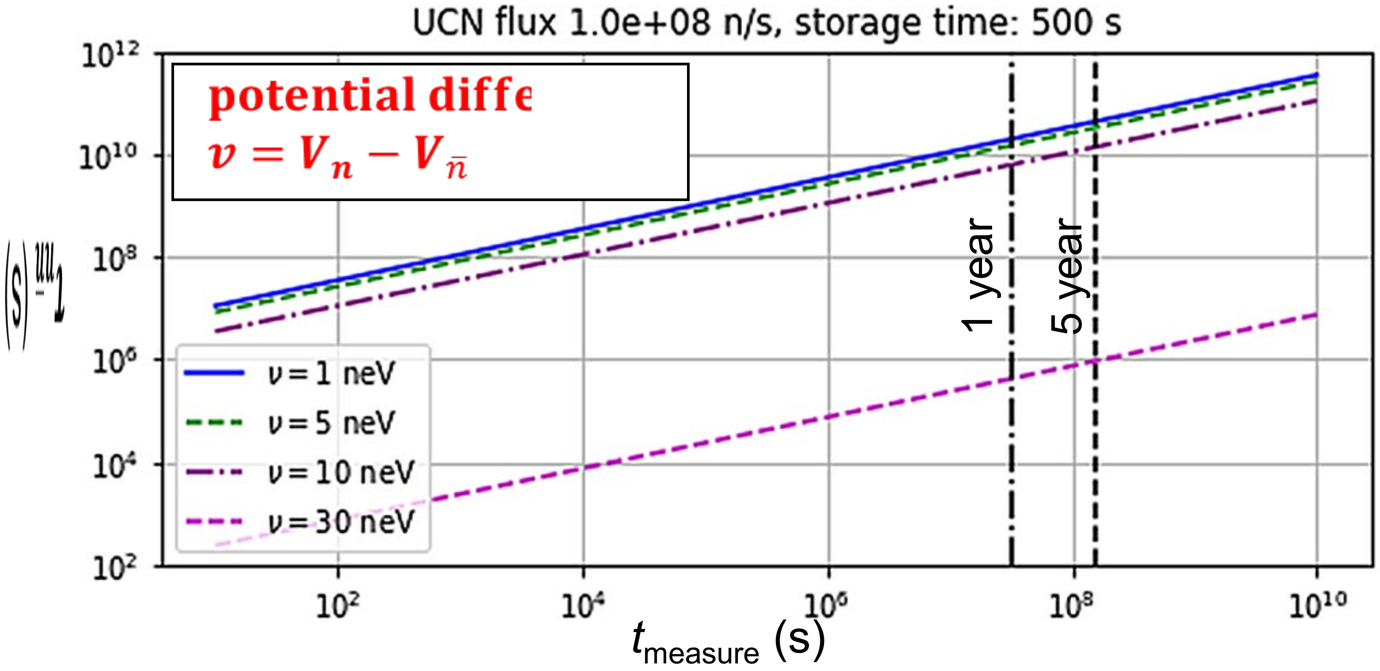


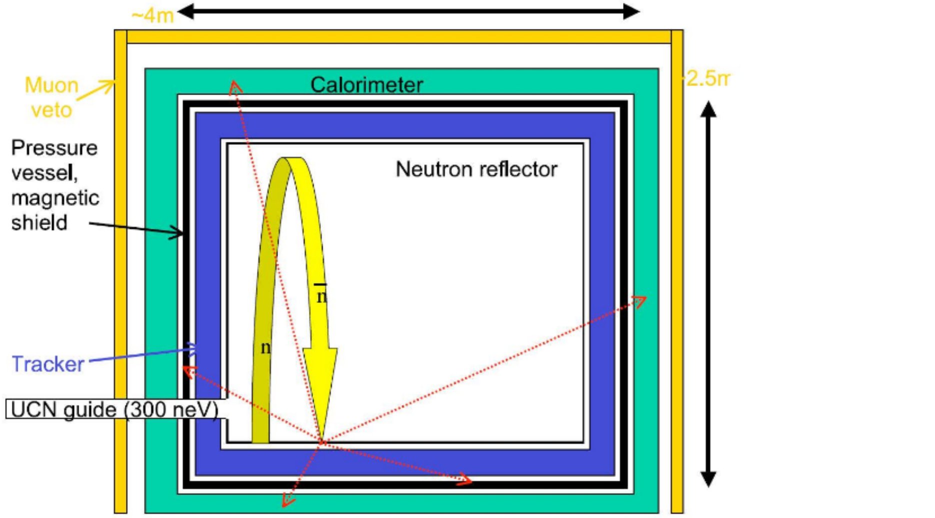

D.G. Phillips II *et al.*, Phys. Rep. **612**, 1-45 (2016)



# Proposed Experiment at the New Research Reactor

- Projected sensitivity: competitive with ESS/HK

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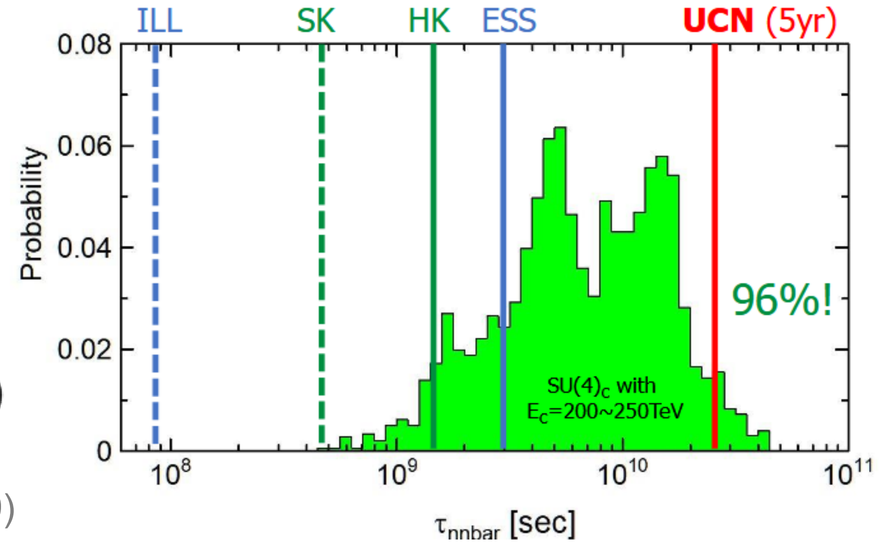


K.S. Babu *et al.*, Phys. Rev. D **87**, 115019 (2013)
T. Shima: NEWS colloquium, RCNP Osaka (2023)

**Challenge**: design of the wall material with a small (~10%) potential difference between $n$ and $\bar{n}$ (i.e. $\bar{n}$-$A$ scattering length)

→ Impacts also on other experiments (e.g. $\bar{n}$ mirror)

V. V. Nesvizhevsky et al. Phys. Rev. Lett., **122**, 221802 (2019)

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   - Uniqueness of the new proposal and the keys
2. Antinucleon-nucleus interactions
   - X-ray spectroscopy of antiprotonic atoms
   - Low-energy antineutron scattering

# Antineutron/antiproton-nucleus interaction: available data

- Theoretical model developed based on pbar atom data
- Theory and experiment can be compared for
  - $\bar{p}$ atom spectroscopy
  - $\bar{p}$ scattering
  - $\bar{p}$ annihilation cross-section
  - $\bar{n}$ annihilation cross-section

**Strategy for $\bar{n}$-nucleus studies:**

- $\bar{p}$ atom spectroscopy (indirect)
- Low-energy $\bar{n}$ scattering (direct)

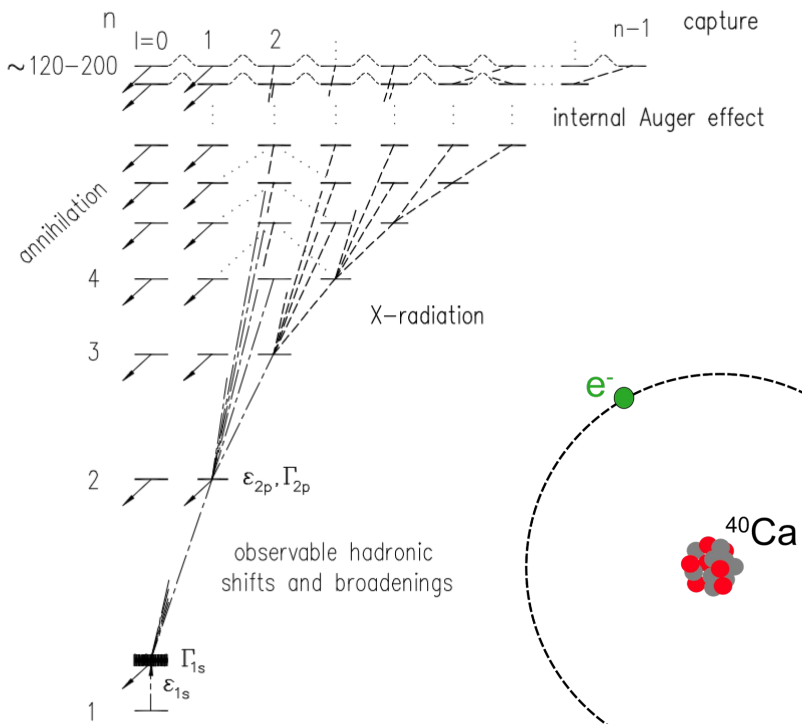
**Table 1** Experimental results for antinucleon-nucleus interaction at low energies

| target | $\bar{p}$ atoms | $\bar{p}$ ann. | $\bar{p}$ scatt. | $\bar{n}$ ann. |
|---|---|---|---|---|
| C | + | | + | + |
| O | + | | | |
| Ne | | + | | |
| Al | | | | + |
| Ca | + | | + | |
| Fe | + | | | |
| Ni | + | + | | |
| Cu | | | | + |
| Zr | + | | | |
| Ag | | | | + |
| Cd | + | | | |
| Sn | + | + | | + |
| Te | + | | | |
| Pt | | + | | |
| Pb | + | | + | + |
| data points | 90 | 7 | 88 | 42 |

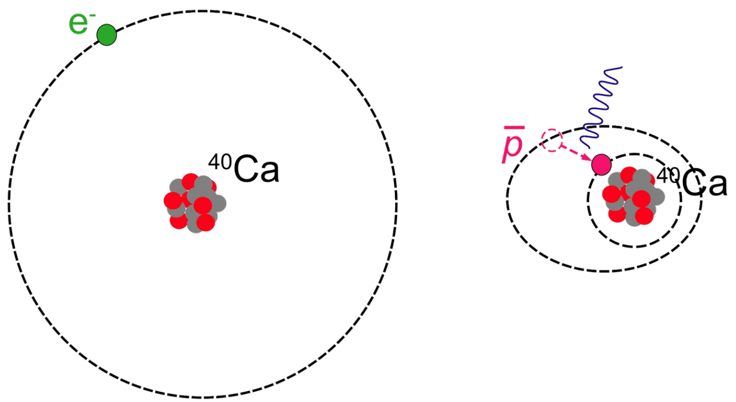
E. Friedman, *Hyperfine Interact.*, **234**, 77–84 (2015).

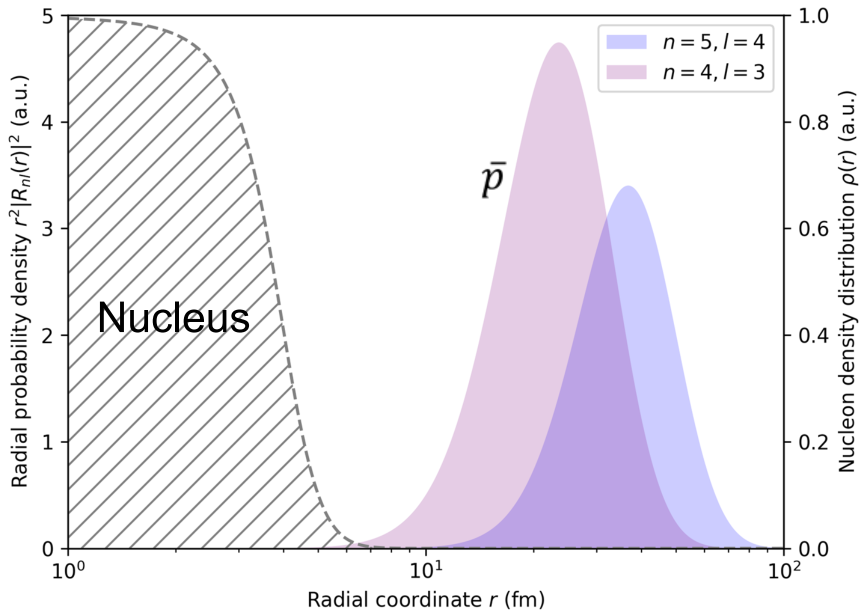
# Antiprotonic atom spectroscopy

- Antiproton captured around $n_{\bar{p}} \sim n_e\sqrt{m_{\bar{p}}/m_e}$, $n \sim l-1$
- Cascade down through Auger transitions ($\Delta n_{\bar{p}} = -1$) and radiative (E1) decays ($\Delta n_{\bar{p}} = -1$, $\Delta l_{\bar{p}} = -1$):
  → Characteristic X-rays: reflect level shifts $\varepsilon = E_{exp} - E_{EM}$ and width $\Gamma$ due to strong interaction




D. Gotta et al., Eur. Phys. J D **47**, 11 (2008)




$$\Delta E_{\text{strong}} = \epsilon - i\,\frac{\Gamma}{2} = \int \Psi_{nl}^{\dagger} V_{\text{opt}} \Psi_{nl} dV$$

C.J. Batty, E. Friedman & A. Gal, Nuc. Phys. A **689**, 283 (2001)
E. Friedman, A. Gal, & J. Mareš, Nucl. Phys. A **761**, 283 (2005)

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**Strong interaction represented by the optical model**

Optical potential
$V_{opt} \propto$ (scattering length per nucleon) x (nucleon density)

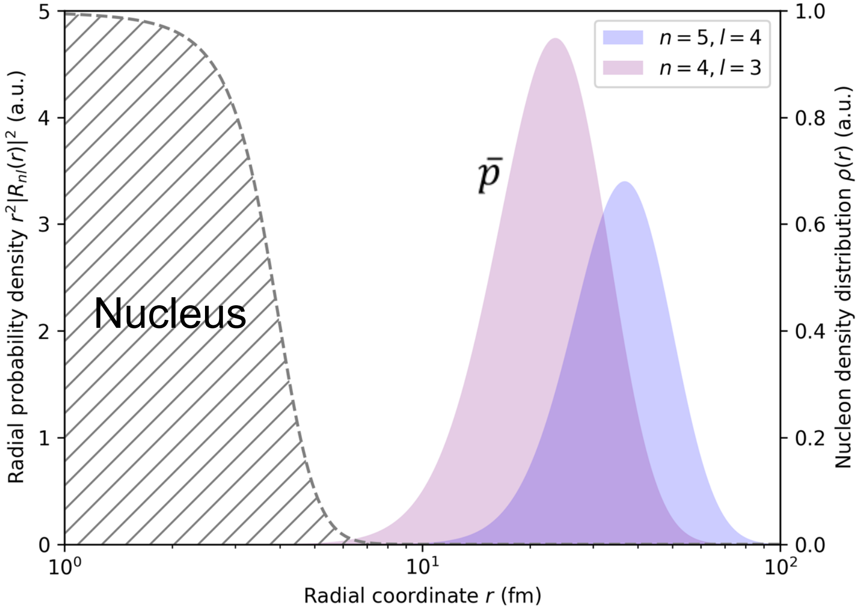
$$2\mu V_{opt}(r) = -4\pi\left(1 + \frac{\mu}{M}\frac{A-1}{A}\right)[b_0(\rho_n + \rho_p) + b_1(\rho_n - \rho_p)]$$

$\mu$ : reduced mass
$\rho_n$, $\rho_p$: nucleon density

Isospin transform $(\bar{n} \to \bar{p})$: $b_1 \to -b_1$

$b_0$ : isoscalar term
$b_1$ : isovector term

C.J. Batty, E. Friedman & A. Gal, Physics Report **287**, 385 (1997).

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# Results of pbar-A scattering lengths from a global fit

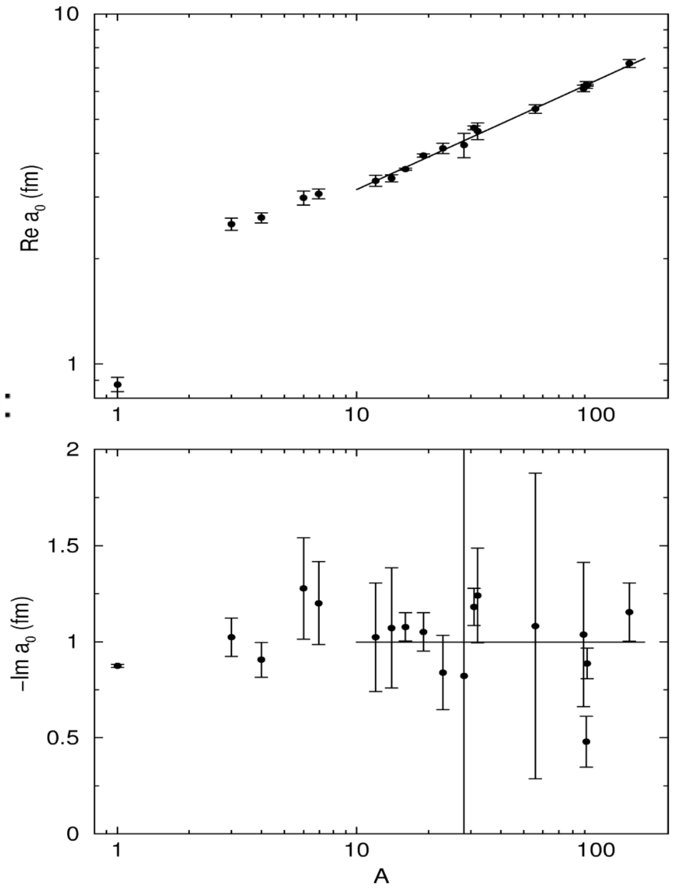
- C.J. Batty et al 2001:
  - Global fit of $\bar{p}$-atom data from C to Pr
  - Scaled with $\propto A^{1/3}$

$$\mathrm{Re}\, a_0 = (1.54 \pm 0.03) A^{0.311 \pm 0.005}\ \mathrm{fm}$$

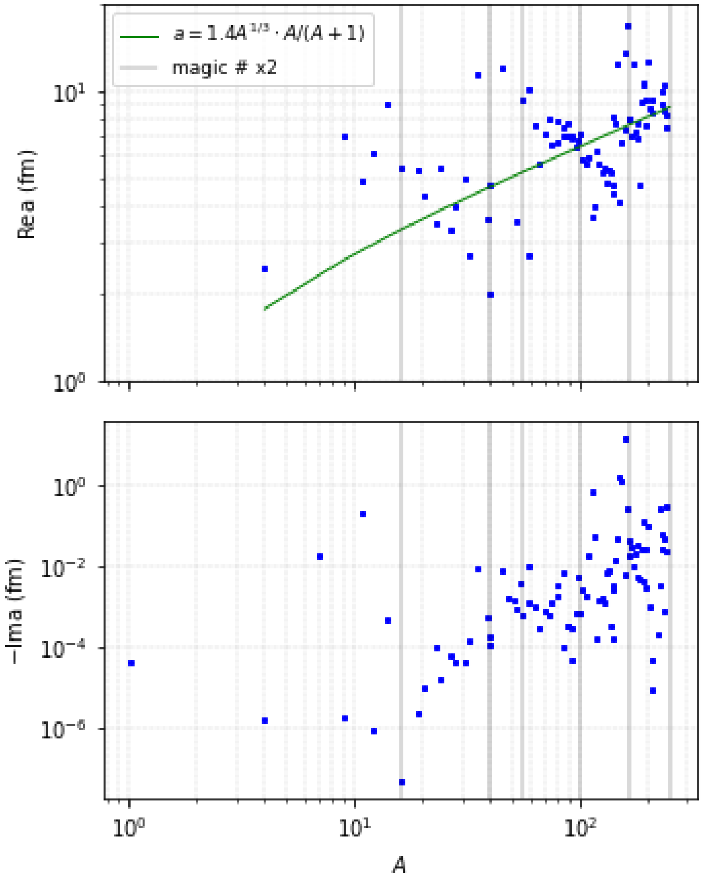
$$\mathrm{Im}\, a_0 = -1.00 \pm 0.04\ \mathrm{fm}.$$

- Compared with neutron-nucleus scattering length:
  - Simpler scaling against the mass number $A$ (implies insensitivity on nuclear shell structure)
  - Orders of magnitudes larger imaginary part (annihilation)
- $\bar{n}$-$A$ optical potential: obtained by an isospin transform ($n \leftrightarrow p$, $b_1 \rightarrow -b_1$), but the isovecter term $b_1$ is poorly constrained ($b_1$=0 assumed)

$\bar{p}$ - A




$n$- A



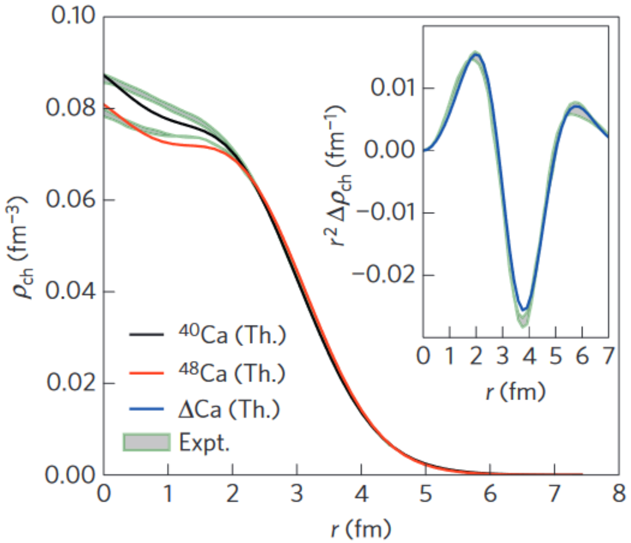

C.J. Batty, E. Friedman & A. Gal, Nucl. Phys. A, 689, 721–740 (2001).
E. Friedman, A. Gal, & J. Mareš, Nucl. Phys. A **761**, 283 (2005).
V.V. Nesvizhevsky, G. Pignol, & K.V. Protasov, *Phys. Rev. D* **77** 034020 (2008).

# New proposal: $\bar{p}$ Ca atom spectroscopy

$$2\mu V_{opt}(r) = -4\pi\left(1 + \frac{\mu}{M}\frac{A-1}{A}\right)\left[b_0(\rho_n + \rho_p) + \boxed{b_1}(\rho_n - \rho_p)\right]$$

- **Purpose: determine the isovector term $b_1$**
- Limitation of previous experiments: knowledge of nuclear density distributions
- **Key 1)** Use calcium (Ca) isotopes:
  - Advanced theoretical & experimental studies on nucleon distribution
  - Long chain of stable isotopes (A=40,42, 44, 48), doubly-magic: 40, 48
  - $^{40}$Ca: N=Z=20 □ isovector contribution canceled. Isotope shifts reveal the contribution of the additional neutrons

    Backed up by theory calculations (K. Yoshimura et al. arXiv: 2408.14760)
  - Narrow width of the target transition suited for precision measurement

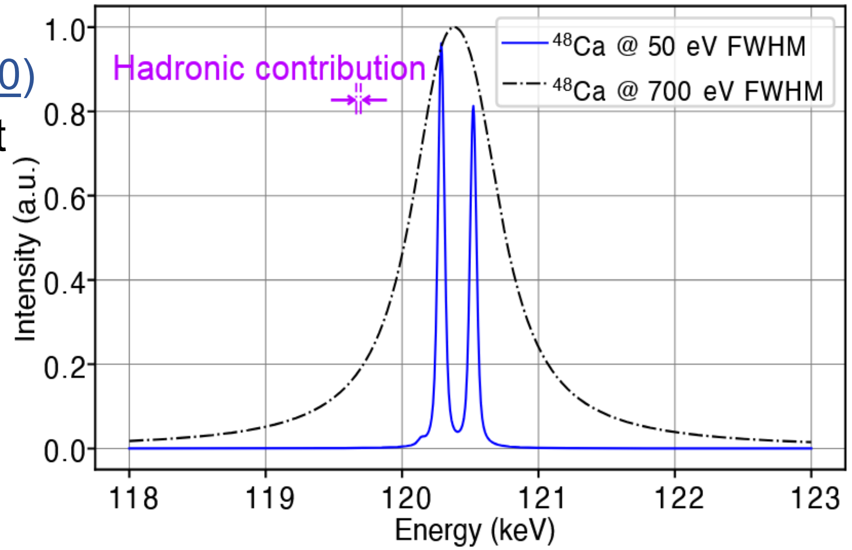



G. Hagen et al., Nat. Physics **12**, 186 (2016)

- **Key 2)** Transition-Edge Sensors (TES) with O(10 eV) resolution
  - Strong interaction shifts with **O(1) eV** precision possible
  - Proposed as a parasitic experiment of the PAX (antiProtonic Aom X-ray Spectroscopy) project

    □ First beamtime at CERN currently ongoing!

PAX
ANTIPROTONIC ATOM X-RAY SPECTROSCOPY




T. Higuchi & H. Fujioka, PoS **049** (2025)

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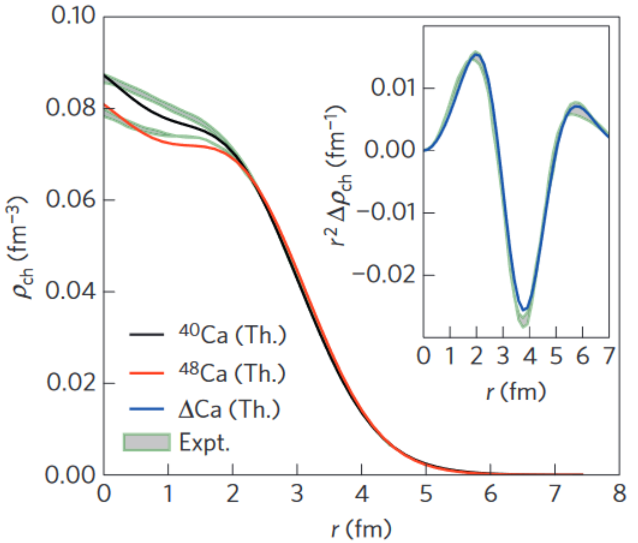
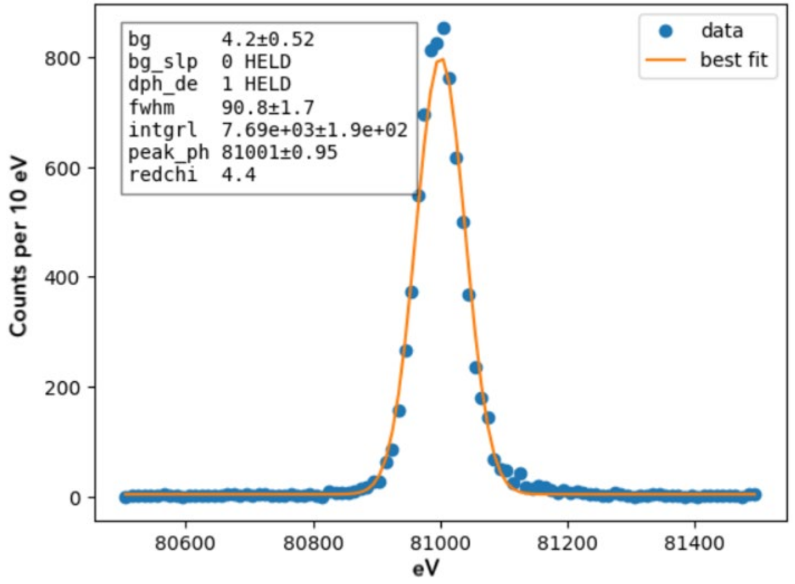
ANTIPROTONIC ATOM X-RAY SPECTROSCOPY




G. Hagen et al., Nat. Physics **12**, 186 (2016)

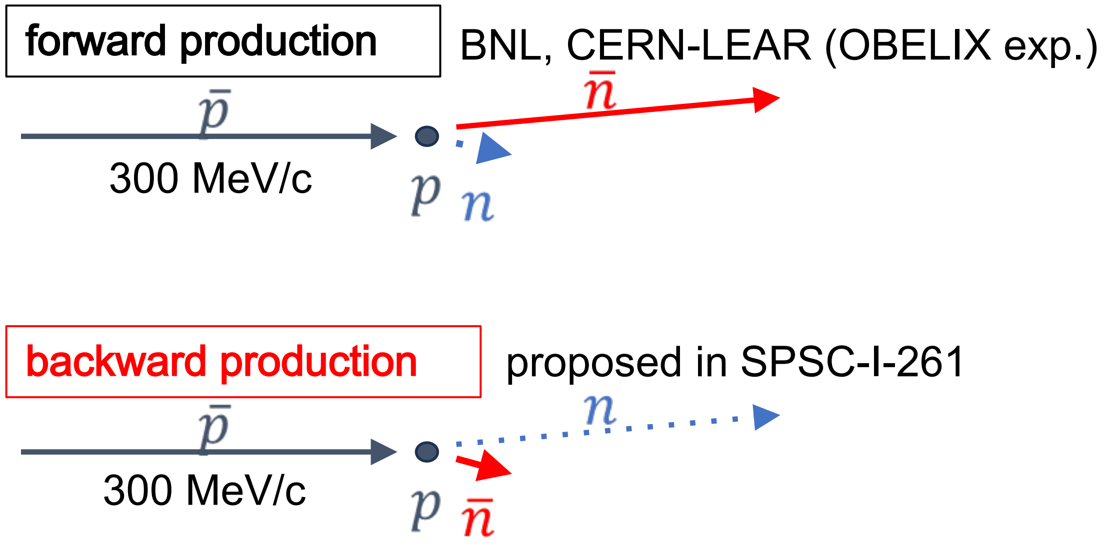


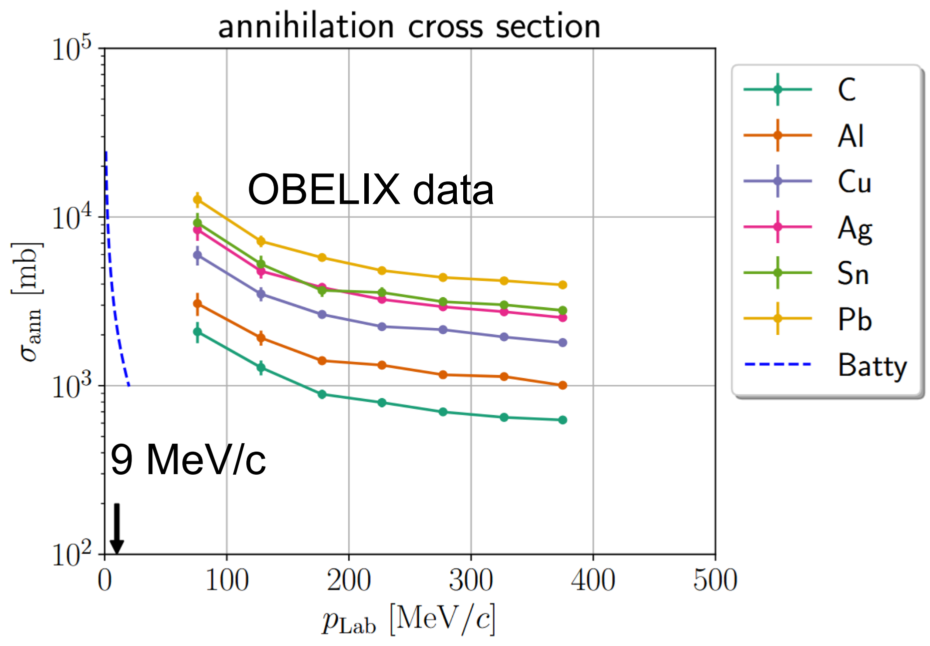

CERN-SPSC-2025-020 (2025)

# Proposal of low-energy antineutron beamline at CERN

- Only existing data in $\bar{n}$-nucleus cross sections: annihilation cross section measurements by OBELIX at > 100 MeV/c (1990). No measurement of elastic scattering
- New Letter of Intent (SPSC-I-261):
  - Low-energy (9 MeV/c, 43 keV) antineutron production by <u>backward</u> charge-exchange reaction $\bar{p}p \to \bar{n}n$ (threshold: 98 MeV/c, 5.1 MeV) using a 300 MeV/c $\bar{p}$ beam from CERN/AD
  - Realize s-wave scattering regime for light nuclei

→ **<u>First direct measurement of $\bar{n}$-nucleus scattering lengths!</u>**

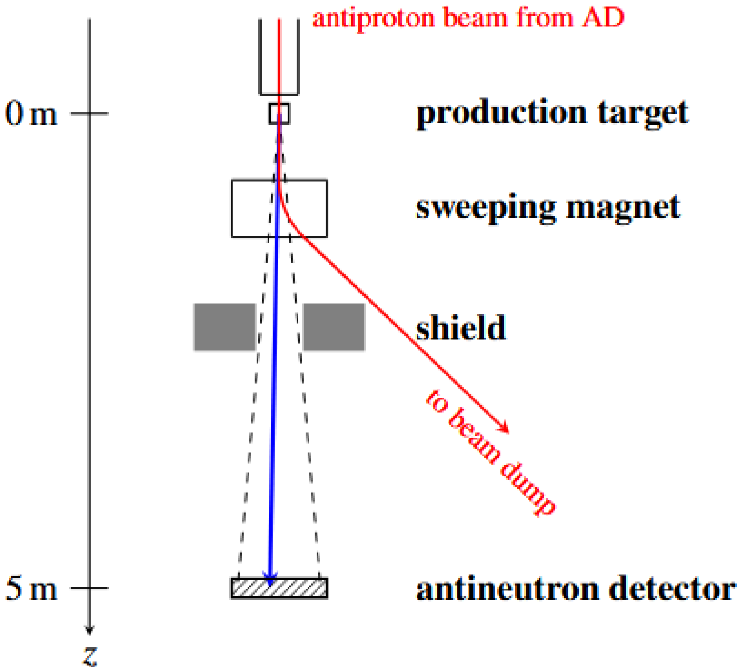


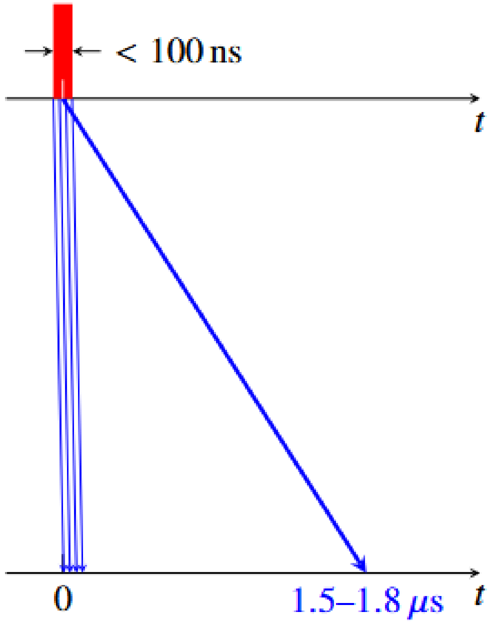




C. Amsler et al., <u>CERN-SPSC-2025-010 (2025)</u>

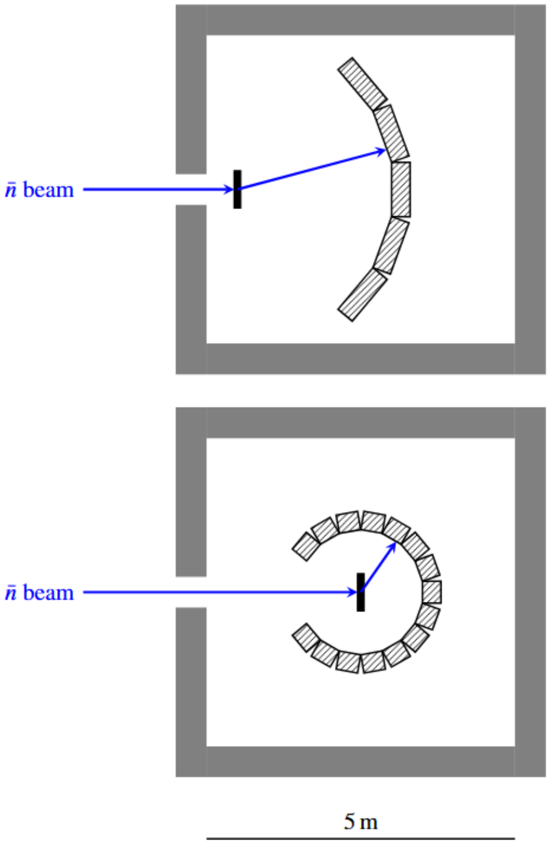
# Proposed future layout and recent/upcoming activities

- Requesting a beamline and an experimental area of ~5 m x 5 m (2030-)
- Potential $\bar{n}$ production test with an existing beamline (2028-)
  - Demonstrate the production principle
  - Study the background events and rates




Expected rate: 1 slow $\bar{n}$ per cycle

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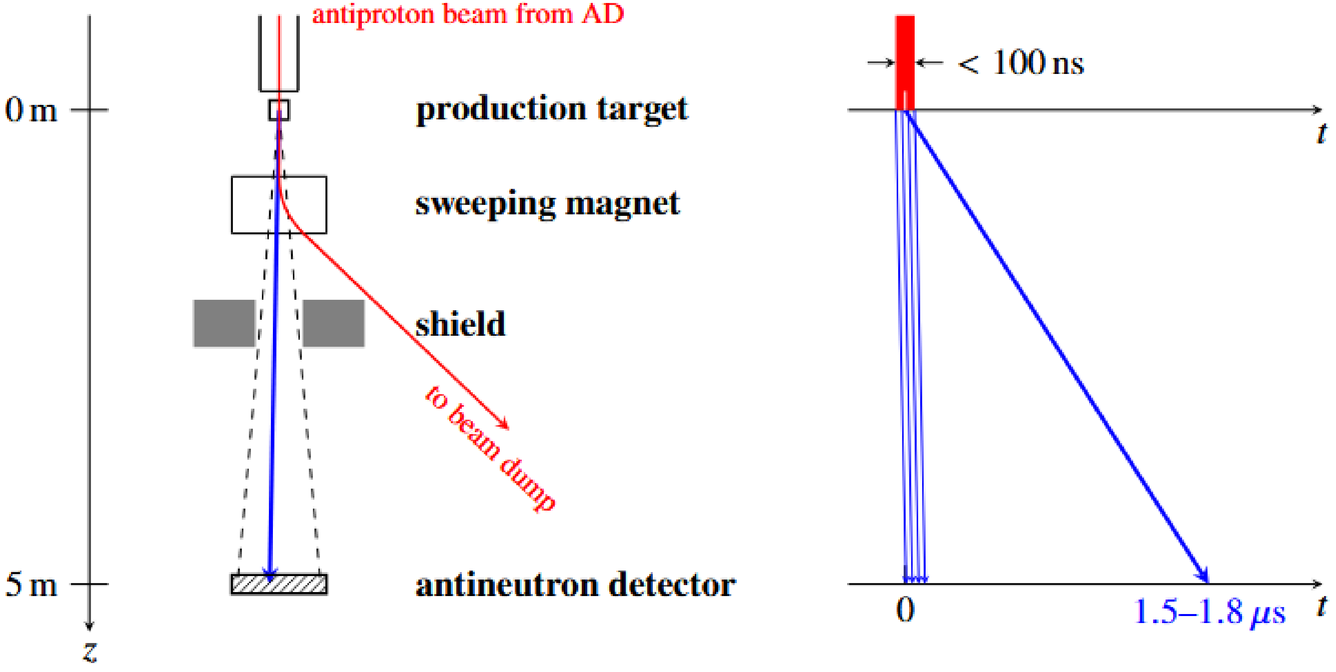
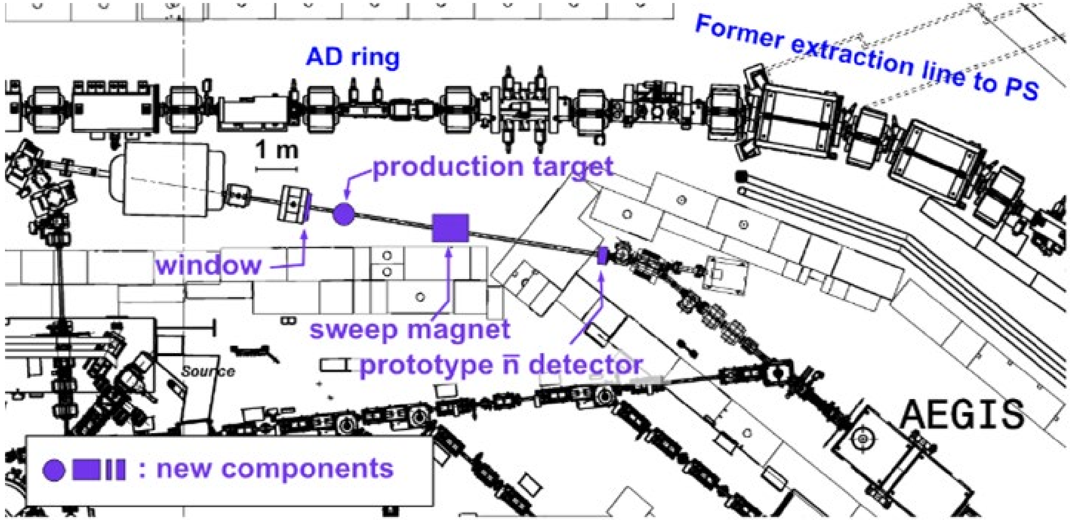
# Proposed future layout and recent/upcoming activities

- Requesting a beamline and an experimental area of ~5 m x 5 m (2030-)
- Potential $\bar{n}$ production test with an existing beamline (2028-)
  - Demonstrate the production principle
  - Study the background events and rates

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# Conclusions

- High-intensity UCN source proposed at the New Research Reactor will provide an opportunity of a search for neutron-antineutron oscillations with unprecedented sensitivity
- Rigorous understanding of the antineutron-nucleus scattering lengths is essential. A series of antinucleon experiments proposed in this context.
  - Antiprotonic atom spectroscopy: isotope shifts of $\bar{p}$ Ca atom X rays with a TES detector
  - Low-energy antineutron scattering: beamline concept developed, the Letter of Intent submitted to CERN

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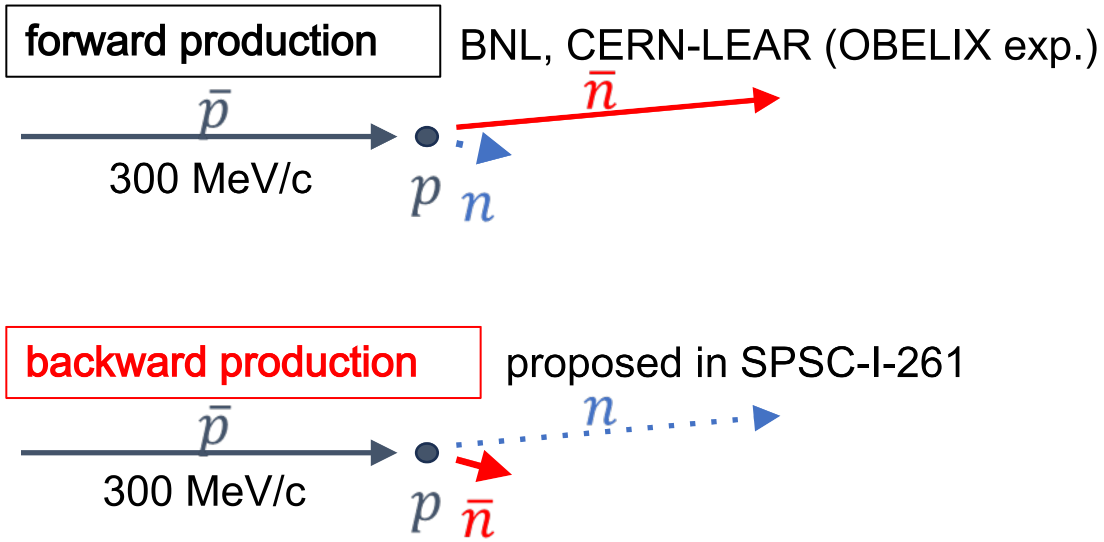
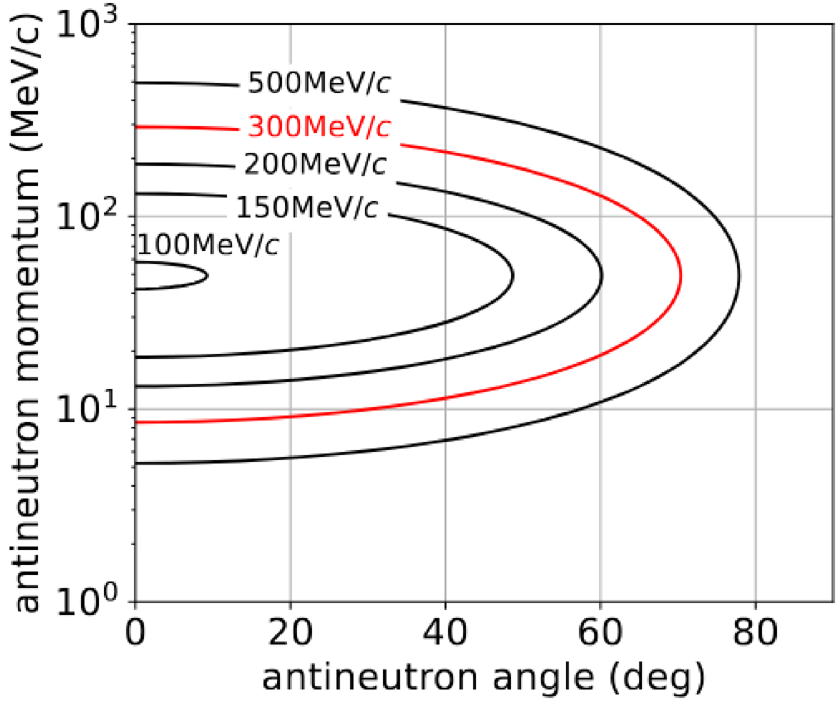
*Thank you for your attention!!*

# Proposal of low-energy antineutron beamline at CERN

- Only existing data in $\bar{n}$-nucleus cross sections: annihilation cross section measurements by OBELIX at > 100 MeV/c (1990). No measurement of elastic scattering
- New Letter of Intent (SPSC-I-261):
  - Low-energy (9 MeV/c, 43 keV) antineutron production by <u>backward</u> charge-exchange reaction $\bar{p}p \to \bar{n}n$ (threshold: 98 MeV/c, 5.1 MeV) using a 300 MeV/c $\bar{p}$ beam from CERN/AD
  - Realize s-wave scattering regime for light nuclei
  - → <u>**First direct measurement of $\bar{n}$-nucleus scattering lengths!**</u>

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# Scattering lengths formula

- Relations of cross sections and scattering lengths
- S-wave dominance: $p \lesssim \hbar/(2R)$ (e.g. $p \lesssim$ 20 MeV/c for Cu)
- Partial-wave expansion ($k = p/\hbar$):

$$f(\theta) = \frac{1}{2ik} \sum_{\ell=0}^{\infty} (2\ell + 1)(S_\ell - 1) P_\ell(\cos\theta)$$

$$\sigma_{\mathrm{el}} = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1)|1 - S_\ell|^2$$

$$\sigma_{\mathrm{ann}} = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1)(1 - |S_\ell|^2)$$

$$\sigma_{\mathrm{tot}} = \frac{2\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1)(1 - \mathrm{Re}\, S_\ell)$$

S-wave approximation (*l*=0)

$$\sigma_{\mathrm{el}} = 4\pi|a|^2(1 - 2a_{\mathrm{I}}k) + O(k^2)$$

$$\sigma_{\mathrm{ann}} = \frac{4\pi}{k} a_{\mathrm{I}} - 8\pi a_{\mathrm{I}}^2 + O(k).$$

$$\sigma_{\mathrm{tot}} = \frac{4\pi}{k} a_{\mathrm{I}} + 4\pi(a_{\mathrm{R}}^2 - a_{\mathrm{I}}^2) + O(k),$$